CALCULATING COEFFICIENTS

This annex outlines a procedure for analysing a fifth order DSM and for calculating coefficients of a desired filter characteristic.

A fifth order DSM is shown in Figure A having coefficients a to f and A to E, adders 6 and integrators 7. Integrators 7 each provide a unit delay. The outputs of the integrators are denoted from left to right s to w. The input to the DSM is a signal x[n] where [n] denotes a sample in a clocked sequence of samples. The input to the quantizer Q is denoted y[n] which is also the output signal of the DSM. The analysis is based on a model of operation which assumes quantizer Q is simply an adder which adds random noise to the processed signal. The quantizer is therefore ignored in this analysis.

The signal y[n] = fx[n] + w[n] i.e. output signal y[n] at sample [n] is the input signal x[n] multiplied by coefficient f plus the output w[n] of the preceding integrator 7.

Applying the same principles to each output signal of the integrators 7 results in Equations set 1.

$$y[n] = fx[n] + w[n]$$

$$w[n] = w[n-1] + ex[n-1] + Ey[n-1] + v[n-1]$$

$$v[n] = v[n-1] + dx[n-1] + Dy[n-1] + u[n-1]$$

$$u[n] = u[n-1] + cx[n-1] + Cy[n-1] + t[n-1]$$

$$t[n] = t[n-1] + bx[n-1] + By[n-1] + s[n-1]$$

$$s[n] = s[n-1]+ax[n-1]+Ay[n-1]$$

These equations are transformed into z-transform equations as well known in the art resulting in equations set 2.

$$Y(z) = fX(z)+W(z)$$

$$W(z)(1-z^{-1}) = z^{-1}(eX(z)+EY(z)+V(z))$$

$$V(z)(1-z^{-1}) = z^{-1}(dX(z)+DY(z)+U(z))$$

$$U(z)(1-z^{-1}) = z^{-1}(cX(z)+CY(z)+T(z))$$

$$T(z)(1-z^{-1}) = z^{-1}(bX(z)+BY(z)+S(z))$$

$$S(z)(1-z^{-1}) = z^{-1}(aX(z)+AY(z))$$

The z transform equations can be solved to derive Y(z) as a single function of X(z) (Equation 3)

$$Y(z) = fX(z) + \frac{z^{-1}}{(1-z^{1})}(eX(z) + EY(z) +$$

$$\frac{z^{-1}}{1-z^{-1}}(dX(z) + DY(z) +$$

$$\frac{z^{-1}}{1-z^{-1}}(cX(z) + CY(z) +$$

$$\frac{z^{-1}}{1-z^{-1}}(bX(z) + BY(z) +$$

$$\frac{z^{-1}}{1-z^{-1}}(aX(z) + AY(z))))))$$

This may be reexpressed as shown in the right hand side of the following equation, Equation 4. A desired transfer function of the DSM can be expressed in series form

$$\frac{Y(z)}{X(z)}$$

given in left hand side of the following equation and equated with the right hand side in Equation 4.

$$\frac{Y(z)}{X(z)} = \frac{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \alpha_3 z^{-3} + \alpha_4 z^{-4} + \alpha_5 z^{-5}}{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} + \beta_3 z^{-3} + \beta_4 z^{-4} + \beta_5 z^{-5}}$$

$$= \frac{f(1-z^{-1})^5 + z^{-1}e(1-z^{-1})^4 + z^{-2}d(1-z^{-1})^3 + z^{-3}c(1-z^{-1})^2 + z^{-4}b(1-z^{-1}) + z^{-5}a}{(1-z^{-1})^5 - z^{-1}E(1-z^{-1})^4 - z^{-2}D(1-x^{-1})^3 - z^{-3}C(1-z^{-1})^2 - z^{-4}B(1-z^{-1}) - Z^{-5}A}$$

Equation 4 can be solved to derive the coefficients f to a from the coefficients α_0 to α_5 and coefficients E to A from the coefficients β_0 to β_5 as follows noting that the coefficients α_n and β_n are chosen in known manner to provide a desired transfer function.

f is the only z^0 term in the numerator. Therefore $f = \alpha_0$.

The term $\alpha_0(1-z^{-1})^5$ is then subtracted from the left hand numerator resulting in

$\alpha_0 + \alpha_1 z^{-1} \ldots + \ldots \alpha_5 z^{-5} - \alpha_0(1-z^{-1})^5$ which is recalculated.

Similarly $f(1-z^{-1})^5$ is subtracted from the right hand numerator. Then e is the only z^{-1} term and can be equated with the corresponding α_1 in the recalculated left hand numerator.

The process is repeated for all the terms in the numerator.

The process is repeated for all the terms in the denominator.